P425/1
PURE MATHEMATICS
Paper 1
July/August 2024
3 hours

WESTERN JOINT MOCK EXAMINATIONS

Uganda Advanced Certificate of Education

PURE MATHEMATICS

Paper 1

3 hours

INSTRUCTIONS TO CANDIDATES:

- *Answer* **all** *the eight questions in Section* **A**, *and any* **five** *from section* **B**.
- *Any additional question(s) attempted will* **not** *be marked.*
- ***All*** *your working* **must** *be clearly shown.*
- *Begin each question on a fresh sheet of paper.*
- *Silent, non-programmable scientific calculators and mathematical tables with a list of formulae may be used.*

SECTION A (40 MARKS)

*Answer **all** the questions in this section*

1. If $\tan x = \frac{7}{24}$ and $\cos y = \frac{-4}{5}$ where x is reflex and y is obtuse, find without using tables or calculators the value of $\sin(x + y)$. *(05 marks)*

2. Calculate the perpendicular distance between the parallel lines $3x + 4y + 10 = 0$ and $3x + 4y - 15 = 0$. *(05 marks)*

3. The sum of n terms of a particular series is given by $S_n = 17n - 3n^2$.
   (a) Find an expression for the n^{th} term of the series.
   (b) Show that the series is an Arithmetic progression. *(05 marks)*

4. Use the calculus of small changes to find $\sin 149.82°$ correct to five significant figures. *(05 marks)*

5. Solve the equation $(\log_3 x^2)(\log_{9x} 3) = 1$ *(05 marks)*

6. Find the coordinates of the point where the line $\frac{x-3}{5} = \frac{3-y}{-2} = \frac{z-4}{3}$ meets the plane $2x - 3y + 7z - 10 = 0$ *(05 marks)*

7. Evaluate $\int_0^4 2x\sqrt{4-x}\,dx$ *(05 marks)*

8. At any point on a Cartesian curve $\frac{dy}{dx} = (3x - 2)(x + 2)$. Given that it passes through the point (1, 1), find the equation of the curve. *(05 marks)*

SECTION B (40 MARKS)

*Answer **five** questions from this section.*

9. (a) Evaluate $\int_0^{½} \frac{x+2}{4x^2+1}\,dx$ *(06 marks)*

   (b) Show that $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{dx}{1-\cos^2 x} = \frac{1}{\sqrt{3}}$ *(06 marks)*

10. (a) A student deposits Shs 1,200,000 once into her Savings account on which an interest of 8% is compounded per annum. After how many years will her balance exceed Shs 2,000,000? *(05 marks)*

    (b) The polynomial $f(x) = x^4 + ax^3 + bx^2 + cx + 1$ is divisible by $(x + 1)^2$ and leaves a remainder of 12 when divided by $(x - 1)$. Find the values of a, b and c. *(07 marks)*

11. (a) **A** and **B** are the points (3, 1, 1) and (5, 2, 3) respectively, and **C** is a point on the line $\underset{\sim}{r} = \begin{pmatrix} 2 \\ 4 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$.

If angle BAC = 90°, find the coordinates of **C**. *(06 marks)*

(b) The vector $\begin{pmatrix} 4 \\ 1 \\ 2 \end{pmatrix}$ is perpendicular to the plane containing the line $\frac{x-3}{-2} = \frac{y+1}{a} = \frac{z-2}{1}$, find the;

(i) value of a.

(ii) Cartesian equation of the plane. *(06 marks)*

12. (a) In a triangle ABC, $\overline{AB} = 10$cm, $\overline{BC} = 17$cm and $\overline{AC} = 21$cm, calculate the angle BAC. *(05 marks)*

(b) Solve the equation $\sin 3x + \sin 7x = \sin 5x$ for $0° \leq x \leq 90°$ *(07 marks)*

13. (a) Solve the equation $2z - i\bar{z} = 5 - i$ where $z = x + iy$ *(05 marks)*

(b) Express the complex number $z = \frac{(3i+1)(i-2)^2}{i-3}$ in the form $a + bi$ where a and b are integers. Hence find

(i) the modulus of z

(ii) the principal argument of z *(07 marks)*

14. (a) The radius of a sphere is decreasing at the rate of $3cms^{-1}$. Obtain the rate of decrease of the surface area of the sphere when the radius is 18cm. (Use $\pi = 3.14$) *(05 marks)*

(b) Differentiate the following with respect to x.

(i) $\log_2 tan^3(4x + 5)$

(ii) $\frac{(3x^2+5)^4}{(2x-3)^3}$ *(07 marks)*

15. (a) Show that $3x^2 + 2y^2 + 6x - 8y = 7$ is an ellipse and hence determine its centre and eccentricity. *(06 marks)*

(b) P is a point on the ellipse whose parametric equation is given by $x = 3\cos\theta$ and $= 2\sin\theta$. The line joining the origin, O to P is produced to Q such that $\overline{OQ} = 2\,\overline{OP}$.

Determine the Cartesian equation of the locus of Q. *(06 marks)*

16. In an agricultural plantation, the properties of the total area that has been destroyed by a bacteria disease is **X**. The rate of destruction of the plantation is proportional to the product of the properties already destroyed and that not yet. It was initially noticed that half of the plantation had been destroyed by the disease and that at this rate another quarter of the plantation would be destroyed in the next 6 hours.

(a) Form a differential relating x and time, t.

(b) Calculate the percentage of the population destroyed 12 hours after the disease was noticed. *(12 marks)*

END